The Generation of Internal Waves by Explosive Volcanic Eruptions

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Recent observations have shown that explosive volcanic eruptions can generate low frequency surface pressure signals that are observable with microbarographs, and can be interpreted as atmospheric internal waves (Baines & Sacks 2013). Some examples from the Caribbean island of Montserrat are shown in Figure 1. Suitably interpreted, it is possible that these observations may give information about the eruption in locations where other observations are not possible or available (such as in remote locations, or at night etc.). This requires quantitative understanding of the link between surface pressure signals and the properties of the eruption, particularly its strength.

Figure 1. Surface barograph records of four observations of explosions of SVH volcano on Montserrat as recorded at station AIRS (a distance of 5 km). The arrows denote the approximate time of onset of the explosions (from Baines & Sacks 2013).

This paper presents a new model of internal wave generation by volcanic eruptions, in which the forcing is governed by plume dynamics. The effect of an explosive volcanic eruption on its environment can be represented by the horizontal flow on the surface of a vertical cylinder that encloses and is centred on the volcanic plume. An explosive volcanic eruption contains hot solid material as well as gases, and a model of plumes with this property has been described by Woods (1988, 2010). For a typical eruption with no crosswind, the flow forced by the plume consists of inflow at low levels where environmental fluid is entrained into the rising plume, and outward flow at upper levels (over a restricted range of heights) due to spreading of the risen fluid around its level of neutral buoyancy. This flow field is time-dependent, but for an explosive eruption it may be approximated by a steady flow field of the form shown that develops rapidly, exists for a period of time of several minutes, and then
abruptly ceases. A schematic diagram of the fully-developed flow field of the eruption is shown in the left-hand part of Figure 2. The corresponding horizontal velocity on a fixed cylinder of radius \( a \) that is close to the source and encloses it is shown on the right. The forcing therefore consists of the horizontal flow at radius \( a \), driven by entrainment and upper level outflow, and \( a \) is assumed to be small enough to disregard any time delay.

Figure 2. Schematic of the horizontal flow field resulting from a volcanic eruption (left) and the corresponding horizontal flow on a nearby surrounding cylinder of fixed radius \( a \). The maximum height reached by the plume is \( z_{\text{max}} \), and the width of the spreading region is here taken as 0.3\( z_{\text{max}} \).

The volcanic plume model of Woods (1988) specifies four variables at the source: the radius \( b_0 \) of the vent, the upward velocity \( w_0 \), the temperature of the ejected mixture, \( T_0 \), and the gas mass fraction, \( n_0 \). These quantities specify the buoyancy flux at the source, \( F_0 \), which gives a measure of the strength of the eruption. The objective is to relate the surface pressure signals to the eruption properties, in particular the height of the eruption and its strength, \( F_0 \). Figure 3 shows an example, where the upper (\( z_{\text{max}} \)) and lower (taken as 0.7\( z_{\text{max}} \)) boundaries of the spreading region are plotted as functions of \( F_0 \) for a range of \( T_0 \) and \( n_0 \).

The equations governing the wave motion are essentially those of incompressible stratified flow. A more complete form of the equations, including compressibility, has been described by Baines & Sacks 2013 (which uses a different form of forcing). The forcing of internal waves in the present model is due to the rate of change of horizontal fluid velocity on the (vertical) surface of the cylinder. Steady flow does not, by itself, generate internal waves, but a time-varying flow does. Further, the steady flow outside the cylinder (notably, the spreading intrusion) is not described by the model, but the time-varying internal waves are.
Figure 3. The upper limit of the plume ($z_{\text{max}}$) as a function of $F_0$ for $b_0 = 50$ m, $w_0 = 200$ m/sec, and a range of values of $n_0$ and $T_0$. The lower boundary of the upper outflow is taken as 0.7$z_{\text{max}}$, based on numerical simulations (Rooney & Devenish 2014).

**Atmospheric stratification profiles.**

The propagation of internal waves depends on the atmospheric profile of the buoyancy frequency $N$, where $N^2 = -\frac{g}{\rho} \frac{d\rho}{dz}$, where $\rho$ is the potential density of the air. Figure 4 shows vertical profiles of values of $N$ as measured over Iceland (mean monthly values are plotted, and the small-scale curves denote the annual cycle). The main feature here is the discontinuity at the tropopause, at a height of ~ 10 km, with a value of $N \approx 0.012$ in the troposphere, and $N \approx 0.021$ in the stratosphere – the stratosphere is more stably stratified. This type of profile is seen over most of the Earth’s surface, including the Caribbean and the neighbourhood of Montserrat, and the tropopause discontinuity in $N$ is the most significant feature that affects internal wave propagation.

Figure 4. Observed values of the Buoyancy frequency $N$ over Iceland. Monthly mean values are plotted, and the small curves denote the annual cycle at each height. The tropopause height is approximately 10 km.
Internal wave propagation.

The dominant and enduring feature of the atmospheric stratification is the discontinuity in $N$ at the tropopause. Internal waves have frequencies $\omega$ in the range $0 < \omega < N$, and plane waves have the interesting property that their phase and group velocities are perpendicular. They propagate at angles to the horizontal that depend on $\omega/N$ (the group velocity is horizontal for $\omega/N \to 0$, and vertical (but zero) for $\omega/N \to 1$), but not on the wavelength. A wave incident on a discontinuity such as the tropopause will be partially reflected from it, and partially transmitted through it, as shown schematically in Figure 5. A theoretical model that describes waves forced by volcanic eruptions (or by something else) must take account of these properties. Waves generated at a particular location may be expected to decrease in amplitude as they propagate away from the source, because of radial spreading, and also because of the vertical propagation of wave energy to the upper atmosphere.

![Figure 5. Internal wave packet of given frequency incident on a discontinuity in buoyancy frequency $N$ (the tropopause) results in a transmitted wave across it and a reflected wave. Here $N_2 > N_1$. Solid arrows denote the direction of the group velocity (and hence, direction of motion of the wave packets), dashed arrows denote the direction of phase propagation. A similar process occurs when a wave is incident on the tropopause from above.](image)

Waves generated by forcing in the stratosphere (i.e. above the tropopause) propagate upwards and downwards. Those with upward group velocity have no impact on surface pressure; those with downward group velocity are partially reflected at the tropopause, and partially transmitted through it to the troposphere, where they have a signal in the surface pressure.
A representative solution.

Some results for a forcing profile that has the form shown in Figure 2 are shown in Figure 6, where the tropopause height is 14 km, and $z_{\text{max}}$ is below the tropopause at 12 km. Here the eruption is represented by the velocity profile at radius $a = 2.5$ km, and which remains constant for 2 minutes, with abrupt onset and turnoff. This is a realistic model for an explosion on Montserrat, for example, which is not particularly strong, so that the plume is confined to the troposphere.

![Figure 6](image_url)

Figure 6. The time series of surface pressure patterns at various distances from the source for a forcing profile resembling that shown in Figure 2, the latter being in the troposphere. Note outward wave propagation, and the maximum amplitude near the source, decreasing outwards.

These curves show persistent oscillations in surface pressure close to the source, with the largest amplitudes occurring at $r = 7$ km ($R = 2.8$) and $r = 12.6$ km ($R = 5.05$). The overall shape and pattern of the curves are generally similar to the observations shown in Figure 1. The model also indicates that stronger explosive eruptions that reach well into the stratosphere have different signatures (not shown here), with weaker signals in surface pressure that have maximum amplitude at greater distances.

Summary

The results show that the plume-based model for forcing is capable of producing the internal waves generated by a wide variety of eruption types, and that these waves can be related to the nature of the eruption (though only one example is shown here). In particular, they can give information about the level of outflow and the strength of the eruption. There is a large variety of possibilities, and work is in progress to clarify the relationship between types of explosive eruptions and the variety of possible internal wave fields that they can produce. The presence of a crosswind has not been considered, but would introduce asymmetry in the forcing (Baines 2013, Rooney & Devenish 2014), as well as in the wave propagation.
References.


